Package 'univariateML'

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Type Package

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Contents

`univariateML-package` *univariateML*

Description

An R-package for fast, easy, and reliable maximum likelihood estimation for a selection of parametric univariate densities.

Details

Data analysis often requires the estimation of univariate densities. Maximum likelihood estimation is sensible for almost every univariate density used in practice. Moreover, the maximum likelihood estimator is typically consistent and efficient.

The purpose of this package is to

- Support maximum likelihood estimation of a large selection of densities.
- Supports plenty of generics such as `plot` and `AIC` to aid your data analysis.

Read the vignettes to learn more about univariateML: `browseVignettes(package = "univariateML")`

Author(s)

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See Also

Useful links:

- https://github.com/JonasMoss/univariateML
- https://jonasmoss.github.io/univariateML/
- Report bugs at https://github.com/JonasMoss/univariateML/issues

`abalone` *Abalone data*

Description

Physical measurements of 4177 abalones, a species of sea snail.

Usage

```
abalone
```

Format

A tibble with 4,177 observations and 9 variables:

sex Sex of the abalone, `F` is female, `M` male, and `I` infant.

length Longest shell measurement.

diameter Diameter perpendicular to length.

height Height with with meat in shell.

whole_weight Grams whole abalone.

shucked_weight Grams weight of meat.

viscera_weight Grams gut weight (after bleeding).

shell_weight Grams after being dried.

rings +1.5 gives the age in years.

Details

See the web page https://archive.ics.uci.edu/ml/datasets/Abalone for more information about the data set.

Source

Dua, D. and Graff, C. (2019). UCI Machine Learning Repository https://archive.ics.uci.edu/ml/. Irvine, CA: University of California, School of Information and Computer Science.

References

Ko, V., Hjort, N. L., & Hobaek Haff, I. (2019). Focused information criteria for copulas. Scandinavian Journal of Statistics.

Examples

```
abalone
```

`bootstrapml` *Parametric Bootstrap on Distributions Fitted with Maximum Likelihood*

Description

The parametric bootstrap is a resampling technique using random variates from a known parametric distribution. In this function the distribution of the random variates is completely determined by the `univariateML` object `object`.

Usage

```
bootstrapml(
  object,
  reps = 1000,
  map = identity,
  reducer = stats::quantile,
  ...
)
```

Arguments

`object`	A `univariateML` object.
`reps`	Positive integer. The number of bootstrap samples.
`map`	A function of the parameters of the `univariateML` object. Defaults to the identity.
`reducer`	A reducer function. Defaults to `stats::quantile` with default argument `probs = c(0.025, 0.975)`.
`...`	Passed to `reducer`.

Details

For each bootstrap iteration a maximum likelihood estimate is calculated using the `ml***` function specified by `object`. The resulting numeric vector is then passed to `map`. The values returned by `map` is collected in an array and the `reducer` is called on each row of the array.

By default the `map` function is the identity and the default `reducer` is the quantile function taking the argument `probs`, which defaults to `c(0.025, 0.975)`. This corresponds to a 95\ basic percentile confidence interval and is also reported by `confint()`

Note: The default confidence intervals are percentile intervals, not empirical intervals. These confidence intervals will in some cases have poor coverage as they are not studentized, see e.g. Carpenter, J., & Bithell, J. (2000).

Value

The transposed map-reduced bootstrap samples.

References

Efron, B., & Tibshirani, R. J. (1994). An introduction to the bootstrap. CRC press.

Carpenter, J., & Bithell, J. (2000). Bootstrap confidence intervals: when, which, what? A practical guide for medical statisticians. Statistics in medicine, 19(9), 1141-1164.

See Also

`confint()` for an application of `bootstrapml`.

Examples

```
set.seed(1)
object <- mlgamma(mtcars$qsec)

## Calculate c(0.025, 0.975) confidence interval for the gamma parameters.
bootstrapml(object)

#            2.5%      97.5%
# shape 68.624945 160.841557
# rate   3.896915   9.089194

## The mean of a gamma distribution is shape/rate. Now we calculate a
## parametric bootstrap confidence interval for the mean with confidence
## limits c(0.05, 0.95)

bootstrapml(object, map = function(x) x[1] / x[2], probs = c(0.05, 0.95))

#       5%      95%
# 17.33962 18.31253

## Print a histogram of the bootstrapped estimates from an exponential.
object <- mlexp(mtcars$qsec)
hist(bootstrapml(object, reducer = identity))
```

`confint.univariateML` *Confidence Intervals for Maximum Likelihood Estimates*

Description

Computes a confidence interval for one or more parameters in a `unvariateML` object.

Usage

```
## S3 method for class 'univariateML'
confint(object, parm = NULL, level = 0.95, Nreps = 1000, ...)
```

Arguments

object	An object of class univariateML.
parm	Vector of strings; the parameters to calculate a confidence interval for. Each parameter must be a member of names(object).
level	The confidence level.
Nreps	Number of bootstrap iterations. Passed to bootstrapml().
...	Additional arguments passed to bootstrapml().

Details

confint.univariateML is a wrapper for bootstrapml() that computes confidence intervals for the main parameters of object. The main parameters of object are the members of names(object). For instance, the main parameters of an object obtained from mlnorm are mean and sd. The confidence intervals are parametric bootstrap percentile intervals with limits (1-level)/2 and 1 - (1-level).

Value

A matrix or vector with columns giving lower and upper confidence limits for each parameter in parm.

See Also

stats::confint() for the generic function and bootstrapml() for the function used to calculate the confidence intervals.

Examples

```
object <- mlinvgauss(airquality$Wind)
confint(object) # 95% confidence interval for mean and shape
confint(object, "mean") # 95% confidence interval for the mean parameter
# confint(object, "variance") # Fails since 'variance isn't a main parameter.
```

---

corbet *Frequencies of butterflies collected in Malaya*

Description

Species abundance data from 1943; a classical application of the logarithmic series distribution.

Usage

```
corbet
```

Format

A vector of size 501 containing integer observations between 1 and 24.

References

Fisher, R. A., Corbet, A. S., & Williams, C. B. (1943). The relation between the number of species and the number of individuals in a random sample of an animal population. The Journal of Animal Ecology, 12(1), 42. https://doi.org/10.2307/1411

Examples

```
corbet
```

---

egypt *Mortality data from ancient Egypt*

Description

Age at death of 141 Roman era Egyptian mummies.

Usage

```
egypt
```

Format

A tibble with 141 observations and 2 variables:

age Age at death.

sex Sex of deceased; 82 males and 49 females.

Details

This data was collected by Spiegelberg (1901) and analyzed by Karl Pearson (1902) in the first volume of Biometrika. It was analyzed by Claeskens & Hjort (2008) and the data is based on their transcription.

References

Spiegelberg, W. (1901). Aegyptische und Griechische Eigennamen aus Mumientiketten der R<U+00F6>mischen Kaiserzeit.

Pearson, K. (1902). On the change in expectation of life in man during a period of circa 2000 years. Biometrika, 1(2), 261-264.

Claeskens, G., & Hjort, N. L. (2008). Model selection and model averaging. Cambridge University Press.

See Also

The source of the data is https://feb.kuleuven.be/public/u0043181/modelselection/datasets/egyptlives_data.txt

Examples

```
egypt
```

`get_start` *Returns appropriate starting value*

Description

Returns appropriate starting value

Usage

```
get_start(default, name, ...)
```

Arguments

`default`	Function to calculate default parameter value.
`name`	Name of default starting value.
`...`	Parameter list containing an element `name`.

Value

Default value if `name` is not present

`inverse_digamma` *Inverse digamma function*

Description

Calculates the inverse digamma function using Newton–Raphson. Works for y > -500. Uses Newton–Raphson with relative tolerance of eps^0.25.

Usage

```
inverse_digamma(y)
```

Arguments

`y`	Values to invert.

Details

The number of iterations are few, 1 for most input values, especially those that are large. The starting value is the lower bound found by Batir (2017).

References

Batir, N. (2017). Inequalities for the inverses of the polygamma functions. arXiv. http://arxiv.org/abs/1705.06547

`MaximumLikelihoodDistribution` *Maximum likelihood estimated distribution*

Description

Density, distribution function, quantile function and random generation for a univariate distribution estimated by maximum likelihood.

Usage

```
dml(x, obj, log = FALSE)

pml(q = q, obj, lower.tail = TRUE, log.p = FALSE)

qml(p = p, obj, lower.tail = TRUE, log.p = FALSE)

rml(n = n, obj)
```

Arguments

`x`, `q`	vector of quantiles.
`obj`	an `univariateML` object.
`log`, `log.p`	logical; if `TRUE`, the probabilities p are gives as `log(p)`.
`lower.tail`	logical; if `TRUE` (default), the probabilities are $P[X \leq x]$ otherwise, $P[X > x]$
`p`	vector of probabilities.
`n`	number of observations. If `length(n) > 1`, the length is taken to be the number required.

Details

`dml` is the density, `pml` is the distribution function, `qml` is the quantile function, and `rml` is the random variable generator.

These functions work like their counterparts in `stats`, e.g. Normal. The `univariateML` object contains both maximum likelihood estimates and the identity of the model these estimates were calculated under. These functions are wrappers around underlying density, distribution, quantile and random generation functions where unknown parameters are filled with the maximum likelihood estimates. See the example.

Value

`dml` gives the density, `pml` gives the distribution function, `qml` gives the quantile function, and `rml` generates random deviates.

Examples

```
## Simple example
obj <- mlnorm(airquality$Wind)
dml(0.5, obj) == dnorm(0.5, mean = obj[1], sd = obj[2])

obj <- mlbetapr(airquality$Wind)

# Plot the logarithm of the beta prime distribution.
plot(function(x) dml(x, obj, log = TRUE),
  from = 0, to = 20,
  main = "Logarithm of Density", ylab = NA, lwd = 2
)
```

---

mlbeta *Beta distribution maximum likelihood estimation*

Description

Uses `stat::nlm` to estimate the parameters of the Beta distribution.

Usage

```
mlbeta(x, na.rm = FALSE, ...)
```

Arguments

`x`	a (non-empty) numeric vector of data values.
`na.rm`	logical. Should missing values be removed?
`...`	Ignored.

Details

For the density function of the Beta distribution see Beta.

For `type`, the option `none` is fastest.

Value

`mlbeta` returns an object of class `univariateML`. This is a named numeric vector with maximum likelihood estimates for `shape1` and `shape2` and the following attributes:

`model`	The name of the model.
`density`	The density associated with the estimates.
`logLik`	The loglikelihood at the maximum.
`support`	The support of the density.
`n`	The number of observations.
`call`	The call as captured my `match.call`

References

Johnson, N. L., Kotz, S. and Balakrishnan, N. (1995) Continuous Univariate Distributions, Volume 2, Chapter 25. Wiley, New York.

See Also

Beta for the Beta density, nlm for the optimizer this function uses.

Examples

```
AIC(mlbeta(USArrests$Rape / 100))
```

---

mlbetapr *Beta prime distribution maximum likelihood estimation*

Description

This function does not estimate the scale parameter for the `BetaPrime` distribution. Transforms the data and uses `stat::nlm` to estimate the parameters of the Beta distribution.

Usage

```
mlbetapr(x, na.rm = FALSE, ...)
```

Arguments

`x`	a (non-empty) numeric vector of data values.
`na.rm`	logical. Should missing values be removed?
`...`	passed to `mlbeta`.

Details

For the density function of the Beta prime distribution see BetaPrime.

For `type`, the option `none` is fastest.

Value

`mlbetapr` returns an object of class `univariateML`. This is a named numeric vector with maximum likelihood estimates for `shape1` and `shape2` and the following attributes:

`model`	The name of the model.
`density`	The density associated with the estimates.
`logLik`	The loglikelihood at the maximum.
`support`	The support of the density.
`n`	The number of observations.
`call`	The call as captured my `match.call`

References

Johnson, N. L., Kotz, S. and Balakrishnan, N. (1995) Continuous Univariate Distributions, Volume 2, Chapter 25. Wiley, New York.

See Also

BetaPrime for the Beta prime density, nlm for the optimizer this function uses, mlbeta for the Beta distribution maximum likelihood estimator.

Examples

```
AIC(mlbetapr(USArrests$Rape))
```

---

mlbinom *Binomial distribution maximum likelihood estimation*

Description

For the density function of the Binomial distribution see Binomial.

Usage

```
mlbinom(x, na.rm = FALSE, ...)
```

Arguments

x	a (non-empty) numeric vector of data values.
na.rm	logical. Should missing values be removed?
...	The arguments size can be specified to only return the ml of prob. reltol is the relative accuracy requested, defaults to .Machine$double.eps^0.25. iterlim is a positive integer specifying the maximum number of iterations to be performed before the program is terminated (defaults to 100).

Details

The estimator computes both the size and prob parameter by default. Be aware that the likelihood will often be unbounded. According to Olkin et al. (1981), the likelihood is unbounded when $\hat{\mu}/\hat{\sigma}^2 \leq 1$, where $\hat{\sigma}^2$ is the biased sample variance. When the likelihood is unbounded,the maximum likelihood estimator can be regarded as a Poisson with lambda parameter equal to the mean of the observation.

When $\hat{\mu}/\hat{\sigma}^2 \leq 1$ and size is not supplied by the user, an error is cast. If size is provided and size < max(x), an error is cast.

The maximum likelihood estimator of size is unstable, and improvements exist. See, e.g., Carroll and Lomard (1985) and DasGupta and Rubin (2005).

Value

`mlbinom` returns an object of class `univariateML`. This is a named numeric vector with maximum likelihood estimates for `size` and `prob` and the following attributes:

`model`	The name of the model.
`density`	The density associated with the estimates.
`logLik`	The loglikelihood at the maximum.
`support`	The support of the density.
`n`	The number of observations.
`call`	The call as captured my `match.call`

References

Olkin, I., Petkau, A. J., & Zidek, J. V. (1981). A comparison of n Estimators for the binomial distribution. Journal of the American Statistical Association, 76(375), 637-642. https://doi.org/10.1080/01621459.1981.10477697

Johnson, N. L., Kemp, A. W., & Kotz, S. (2005). Univariate Discrete Distributions (3rd ed.). Wiley-Blackwell.

Carroll, R. J., & Lombard, F. (1985). A Note on N Estimators for the Binomial Distribution. Journal of the American Statistical Association, 80(390), 423-426. https://doi.org/10.1080/01621459.1985.10478134

DasGupta, A., & Rubin, H. (2005). Estimation of binomial parameters when both n,p are unknown. Journal of Statistical Planning and Inference, 130(1-2), 391-404. https://doi.org/10.1016/j.jspi.2004.02.019

See Also

Binomial for the density.

Examples

```
# The likelihood will often be unbounded.
## Not run:
mlbinom(ChickWeight$weight)

## End(Not run)
# Provide a size
mlbinom(ChickWeight$weight, size = 400)

# Or use mlpoiss, the limiting likelihood of the binomial.
mlpois(ChickWeight$weight)
```

mlburr *Burr distribution maximum likelihood estimation*

Description

The maximum likelihood estimator fails to exist when the data contains no values strictly smaller than 1. Then the likelihood converges to the likelihood of the Pareto distribution in this case.

Usage

```
mlburr(x, na.rm = FALSE, ...)
```

Arguments

x	a (non-empty) numeric vector of data values.
na.rm	logical. Should missing values be removed?
...	currently affects nothing.

Details

This function estimates the only the shape parameters of the Burr distribution. The shape is set to 1.

For the density function of the Burr distribution see Burr.

Value

mlburr returns an object of class univariateML. This is a named numeric vector with maximum likelihood estimates for shape1 and shape2 and the following attributes:

model	The name of the model.
density	The density associated with the estimates.
logLik	The loglikelihood at the maximum.
support	The support of the density.
n	The number of observations.
call	The call as captured my match.call

References

Johnson, N. L., Kotz, S. and Balakrishnan, N. (1995) Continuous Univariate Distributions, Volume 1, Chapter 20. Wiley, New York.

See Also

Burr for the Burr density.

Examples

```
mlburr(abalone$length)
```

`mlcauchy` *Cauchy distribution maximum likelihood estimation*

Description

Calculates the estimates using `nlm` and an exponential transform of the location parameter. If `n < 5`, an exact solution is reported. In the edge case where no maximum likelihood estimator exists and error is thrown.

Usage

```
mlcauchy(x, na.rm = FALSE, ...)
```

Arguments

`x`	a (non-empty) numeric vector of data values.
`na.rm`	logical. Should missing values be removed?
`...`	currently affects nothing.

Details

For the density function of the Cauchy distribution see Cauchy.

Value

`mlcauchy` returns an object of class `univariateML`. This is a named numeric vector with maximum likelihood estimates for `location` and `scale` and the following attributes:

`model`	The name of the model.
`density`	The density associated with the estimates.
`logLik`	The loglikelihood at the maximum.
`support`	The support of the density.
`n`	The number of observations.
`call`	The call as captured my `match.call`

References

Johnson, N. L., Kotz, S. and Balakrishnan, N. (1995) Continuous Univariate Distributions, Volume 1, Chapter 16. Wiley, New York.

See Also

Cauchy for the Cauchy density, nlm for the optimizer this function uses.

Examples

```
mlcauchy(airquality$Temp)
```

`mldunif` *Discrete uniform distribution maximum likelihood estimation*

Description

For the density function of the Discrete uniform distribution see DiscreteUniform.

Usage

```
mldunif(x, na.rm = FALSE, ...)
```

Arguments

`x`	a (non-empty) numeric vector of data values.
`na.rm`	logical. Should missing values be removed?
`...`	not in use.

Value

`mldunif` returns an object of class `univariateML`. This is a named numeric vector with maximum likelihood estimates for `min` and `max` and the following attributes:

`model`	The name of the model.
`density`	The density associated with the estimates.
`logLik`	The loglikelihood at the maximum.
`support`	The support of the density.
`n`	The number of observations.
`call`	The call as captured my `match.call`

References

Johnson, N. L., Kemp, A. W., & Kotz, S. (2005). Univariate Discrete Distributions (3rd ed.). Wiley-Blackwell.

See Also

DiscreteUniform for the density.

Examples

```
mldunif(corbet)
```

`mlexp` *Exponential distribution maximum likelihood estimation*

Description

The maximum likelihood estimate of `rate` is the inverse sample mean.

Usage

```
mlexp(x, na.rm = FALSE, ...)
```

Arguments

`x`	a (non-empty) numeric vector of data values.
`na.rm`	logical. Should missing values be removed? If `FALSE`, the function fails when `x` contains missing values.
`...`	currently affects nothing.

Details

For the density function of the exponential distribution see Exponential.

Value

`mlexp` returns an object of class `univariateML`. This is a named numeric vector with maximum likelihood estimates for `rate` and the following attributes:

`model`	The name of the model.
`density`	The density associated with the estimates.
`logLik`	The loglikelihood at the maximum.
`support`	The support of the density.
`n`	The number of observations.
`call`	The call as captured by `match.call`
`continuous`	Is the density continuous or discrete?

References

Johnson, N. L., Kotz, S. and Balakrishnan, N. (1995) Continuous Univariate Distributions, Volume 1, Chapter 19. Wiley, New York.

See Also

Exponential for the exponential density.

Examples

```
mlexp(precip)
```

`mlfatigue` *Gamma distribution maximum likelihood estimation*

Description

Uses Newton-Raphson to estimate the parameters of the Birnbaum–Saunders distribution. The parameter `mu` is set to `0`, hence only `alpha` and `beta` are estmated.

Usage

```
mlfatigue(x, na.rm = FALSE, ...)
```

Arguments

`x`	a (non-empty) numeric vector of data values.
`na.rm`	logical. Should missing values be removed?
`...`	`reltol` is the relative accuracy requested, defaults to `.Machine$double.eps^0.25`. `iterlim` is a positive integer specifying the maximum number of iterations to be performed before the program is terminated (defaults to `100`).

Details

For the density function of the Birnbaum–Saunders distribution see BirnbaumSaunders.

Value

`mlfatigue` returns an object of class `univariateML`. This is a named numeric vector with maximum likelihood estimates for `alpha` and `beta`, with `mu=0`, and the following attributes:

`model`	The name of the model.
`density`	The density associated with the estimates.
`logLik`	The loglikelihood at the maximum.
`support`	The support of the density.
`n`	The number of observations.
`call`	The call as captured my `match.call`

References

From, S. G., & Li, L. (2006). Estimation of the parameters of the Birnbaum<U+2013>Saunders distribution. Communications in Statistics: Theory and Methods, 35(12), 2157<U+2013>2169. https://doi.org/10.1080/03610920600853563

Examples

```
mlfatigue(precip)
```

`mlgamma` *Gamma distribution maximum likelihood estimation*

Description

Uses Newton-Raphson to estimate the parameters of the Gamma distribution.

Usage

```
mlgamma(x, na.rm = FALSE, ...)
```

Arguments

`x`	a (non-empty) numeric vector of data values.
`na.rm`	logical. Should missing values be removed?
`...`	`reltol` is the relative accuracy requested, defaults to `.Machine$double.eps^0.25`. `iterlim` is a positive integer specifying the maximum number of iterations to be performed before the program is terminated (defaults to `100`).

Details

For the density function of the Gamma distribution see GammaDist.

Value

`mlgamma` returns an object of class `univariateML`. This is a named numeric vector with maximum likelihood estimates for `shape` and `rate` and the following attributes:

`model`	The name of the model.
`density`	The density associated with the estimates.
`logLik`	The loglikelihood at the maximum.
`support`	The support of the density.
`n`	The number of observations.
`call`	The call as captured my `match.call`

References

Choi, S. C, and R. Wette. "Maximum likelihood estimation of the parameters of the gamma distribution and their bias." Technometrics 11.4 (1969): 683-690.

Johnson, N. L., Kotz, S. and Balakrishnan, N. (1995) Continuous Univariate Distributions, Volume 1, Chapter 17. Wiley, New York.

See Also

GammaDist for the Gamma density.

Examples

```
mlgamma(precip)
```

`mlged` *Generalized Error distribution maximum likelihood estimation*

Description

Joint maximum likelihood estimation as implemented by fGarch::gedFit.

Usage

```
mlged(x, na.rm = FALSE, ...)
```

Arguments

`x`	a (non-empty) numeric vector of data values.
`na.rm`	logical. Should missing values be removed?
`...`	currently affects nothing.

Details

For the density function of the Student t-distribution see ged.

Value

`mlged` returns an object of class `univariateML`. This is a named numeric vector with maximum likelihood estimates for the parameters `mean`, `sd`, `nu` and the following attributes:

`model`	The name of the model.
`density`	The density associated with the estimates.
`logLik`	The loglikelihood at the maximum.
`support`	The support of the density.
`n`	The number of observations.
`call`	The call as captured my `match.call`

References

Nelson D.B. (1991); Conditional Heteroscedasticity in Asset Returns: A New Approach, Econometrica, 59, 347<U+2013>370.

Fernandez C., Steel M.F.J. (2000); On Bayesian Modelling of Fat Tails and Skewness, Preprint.

See Also

ged for the Student t-density.

Examples

```
mlged(precip)
```

`mlgeom` *Geometric distribution maximum likelihood estimation*

Description

For the density function of the Geometric distribution see Geometric.

Usage

```
mlgeom(x, na.rm = FALSE, ...)
```

Arguments

`x`	a (non-empty) numeric vector of data values.
`na.rm`	logical. Should missing values be removed?
`...`	not in use.

Value

`mlgeom` returns an object of class `univariateML`. This is a named numeric vector with maximum likelihood estimates for `prob` and the following attributes:

`model`	The name of the model.
`density`	The density associated with the estimates.
`logLik`	The loglikelihood at the maximum.
`support`	The support of the density.
`n`	The number of observations.
`call`	The call as captured my `match.call`

References

Johnson, N. L., Kemp, A. W., & Kotz, S. (2005). Univariate Discrete Distributions (3rd ed.). Wiley-Blackwell.

See Also

Geometric for the density.

Examples

```
mlgeom(corbet)
```

`mlgompertz` *Gompertz distribution maximum likelihood estimation*

Description

Uses Newton-Raphson to estimate the parameters of the Gompertz distribution.

Usage

```
mlgompertz(x, na.rm = FALSE, ...)
```

Arguments

`x`	a (non-empty) numeric vector of data values.
`na.rm`	logical. Should missing values be removed?
`...`	`reltol` is the relative accuracy requested, defaults to `.Machine$double.eps^0.25`. `iterlim` is a positive integer specifying the maximum number of iterations to be performed before the program is terminated (defaults to `100`).

Details

For some data sets the maximum likelihood estimator of `b` fails to exist since the root of the profile maximum likelihood equation is non-positive. The value `1e-06` is returned in this case, along with a warning.

For the density function of the Gompertz distribution see Gompertz.

Value

`mlgompertz` returns an object of class `univariateML`. This is a named numeric vector with maximum likelihood estimates for `a` and `b` and the following attributes:

`model`	The name of the model.
`density`	The density associated with the estimates.
`logLik`	The loglikelihood at the maximum.
`support`	The support of the density.
`n`	The number of observations.
`call`	The call as captured my `match.call`

References

Lenart, A. (2012). The Gompertz distribution and Maximum Likelihood Estimation of its parameters - a revision. MPIDR WORKING PAPER WP 2012-008. https://www.demogr.mpg.de/papers/working/wp-2012-008.pdf

See Also

Gompertz for the Gompertz density.

Examples

```
mlgompertz(precip)
```

`mlgumbel` *Gumbel distribution maximum likelihood estimation*

Description

Uses Newton-Raphson to estimate the parameters of the Gumbel distribution.

Usage

```
mlgumbel(x, na.rm = FALSE, ...)
```

Arguments

`x`	a (non-empty) numeric vector of data values.
`na.rm`	logical. Should missing values be removed?
`...`	`reltol` is the relative accuracy requested, defaults to `.Machine$double.eps^0.25`. `iterlim` is a positive integer specifying the maximum number of iterations to be performed before the program is terminated (defaults to `100`).

Details

For the density function of the Gumbel distribution see Gumbel.

Value

`mlgumbel` returns an object of class `univariateML`. This is a named numeric vector with maximum likelihood estimates for `mu` and `s` and the following attributes:

`model`	The name of the model.
`density`	The density associated with the estimates.
`logLik`	The loglikelihood at the maximum.
`support`	The support of the density.
`n`	The number of observations.
`call`	The call as captured my `match.call`

`shape` and `sigma`.

References

Johnson, N. L., Kotz, S. and Balakrishnan, N. (1995) Continuous Univariate Distributions, Volume 2, Chapter 22. Wiley, New York.

See Also

Gumbel for the Gumbel density.

Examples

```
mlgumbel(precip)
```

`mlinvburr` *Inverse Burr distribution maximum likelihood estimation*

Description

The maximum likelihood estimator fails to exist when the data contains no values strictly greater than 1. Then the likelihood converges to the likelihood of the Pareto distribution in this case.

Usage

```
mlinvburr(x, na.rm = FALSE, ...)
```

Arguments

`x`	a (non-empty) numeric vector of data values.
`na.rm`	logical. Should missing values be removed?
`...`	currently affects nothing.

Details

`mlinvburr(x)` calls `mlburr(1/x)` internally.

For the density function of the Inverse Burr distribution see Inverse Burr.

Value

`mlburr` returns an object of class `univariateML`. This is a named numeric vector with maximum likelihood estimates for `shape1` and `shape2` and the following attributes:

`model`	The name of the model.
`density`	The density associated with the estimates.
`logLik`	The loglikelihood at the maximum.
`support`	The support of the density.
`n`	The number of observations.
`call`	The call as captured my `match.call`

References

Johnson, N. L., Kotz, S. and Balakrishnan, N. (1995) Continuous Univariate Distributions, Volume 1, Chapter 20. Wiley, New York.

See Also

Inverse Burr for the Inverse Burr density.

Examples

```
mlburr(abalone$length)
```

`mlinvgamma` *Inverse Gamma distribution maximum likelihood estimation*

Description

Transforms the data and uses Newton-Raphson to estimate the parameters of the Gamma distribution.

Usage

```
mlinvgamma(x, na.rm = FALSE, ...)
```

Arguments

`x`	a (non-empty) numeric vector of data values.
`na.rm`	logical. Should missing values be removed?
`...`	passed to `mlgamma`.

Details

For the density function of the inverse Gamma distribution see InvGamma.

Value

A named numeric vector with maximum likelihood estimates for `alpha` and `beta`.

References

Choi, S. C, and R. Wette. "Maximum likelihood estimation of the parameters of the gamma distribution and their bias." Technometrics 11.4 (1969): 683-690.

Johnson, N. L., Kotz, S. and Balakrishnan, N. (1995) Continuous Univariate Distributions, Volume 1, Chapter 17. Wiley, New York.

Witkovsky, V. (2001). "Computing the Distribution of a Linear Combination of Inverted Gamma Variables". Kybernetika. 37 (1): 79<U+2013>90

See Also

InvGamma for the Inverse Gamma density.

Examples

```
mlinvgamma(precip)
```

`mlinvgauss` *Inverse Gaussian (Wald) maximum likelihood estimation*

Description

The maximum likelihood estimate of `mean` is the empirical mean and the maximum likelihood estimate of `1/shape` is the difference between the mean of reciprocals and the reciprocal of the mean.

Usage

```
mlinvgauss(x, na.rm = FALSE, ...)
```

Arguments

`x`	a (non-empty) numeric vector of data values.
`na.rm`	logical. Should missing values be removed?
`...`	currently affects nothing.

Details

For the density function of the Inverse Gamma distribution see InverseGaussian.

Value

`mlinvgauss` returns an object of class `univariateML`. This is a named numeric vector with maximum likelihood estimates for `mean` and `shape` and the following attributes:

`model`	The name of the model.
`density`	The density associated with the estimates.
`logLik`	The loglikelihood at the maximum.
`support`	The support of the density.
`n`	The number of observations.
`call`	The call as captured my `match.call`

References

Johnson, N. L., Kotz, S. and Balakrishnan, N. (1995) Continuous Univariate Distributions, Volume 1, Chapter 15. Wiley, New York.

See Also

InverseGaussian for the Inverse Gaussian density.

Examples

```
mlinvgauss(precip)
```

`mlinvweibull` *Inverse Weibull distribution maximum likelihood estimation*

Description

The maximum likelihood estimate of `shape` and `rate` are calculated by calling `mlweibull` on the transformed data.

Usage

```
mlinvweibull(x, na.rm = FALSE, ...)
```

Arguments

`x`	a (non-empty) numeric vector of data values.
`na.rm`	logical. Should missing values be removed?
`...`	passed to `mlweibull`.

Details

For the density function of the log normal distribution see InverseWeibull.

Value

`mlinvweibull` returns an object of class `univariateML`. This is a named numeric vector with maximum likelihood estimates for `shape` and `rate` and the following attributes:

`model`	The name of the model.
`density`	The density associated with the estimates.
`logLik`	The loglikelihood at the maximum.
`support`	The support of the density.
`n`	The number of observations.
`call`	The call as captured my `match.call`

References

Kleiber, C. and Kotz, S. (2003), Statistical Size Distributions in Economics and Actuarial Sciences, Wiley.

Klugman, S. A., Panjer, H. H. and Willmot, G. E. (2012), Loss Models, From Data to Decisions, Fourth Edition, Wiley.

Dutang, C., Goulet, V., & Pigeon, M. (2008). actuar: An R package for actuarial science. Journal of Statistical Software, 25(7), 1-37.

See Also

InverseWeibull for the Inverse Weibull density.

Examples

```
mlinvweibull(precip)
```

`mlkumar` *Kumaraswamy distribution maximum likelihood estimation*

Description

Uses Newton-Raphson to estimate the parameters of the Kumaraswamy distribution.

Usage

```
mlkumar(x, na.rm = FALSE, ...)
```

Arguments

`x`	a (non-empty) numeric vector of data values.
`na.rm`	logical. Should missing values be removed?
`...`	`a0` is an optional starting value for the `a` parameter. `reltol` is the relative accuracy requested, defaults to `.Machine$double.eps^0.25`. `iterlim` is a positive integer specifying the maximum number of iterations to be performed before the program is terminated (defaults to `100`).

Details

For the density function of the Kumaraswamy distribution see Kumaraswamy.

Value

mlkumar returns an object of class univariateML. This is a named numeric vector with maximum likelihood estimates for a and b and the following attributes:

model	The name of the model.
density	The density associated with the estimates.
logLik	The loglikelihood at the maximum.
support	The support of the density.
n	The number of observations.
call	The call as captured my match.call

References

Jones, M. C. "Kumaraswamy's distribution: A beta-type distribution with some tractability advantages." Statistical Methodology 6.1 (2009): 70-81.

Kumaraswamy, Ponnambalam. "A generalized probability density function for double-bounded random processes." Journal of Hydrology 46.1-2 (1980): 79-88.

See Also

Kumaraswamy for the Kumaraswamy density.

Examples

```
AIC(mlkumar(USArrests$Rape / 100))
```

---

mllaplace *Laplace distribution maximum likelihood estimation*

Description

The maximum likelihood estimate of mu is the sample median while the maximum likelihood estimate of sigma is mean absolute deviation from the median.

Usage

```
mllaplace(x, na.rm = FALSE, ...)
```

Arguments

x	a (non-empty) numeric vector of data values.
na.rm	logical. Should missing values be removed?
...	currently affects nothing.

Details

For the density function of the Laplace distribution see Laplace.

Value

`mllaplace` returns an object of class `univariateML`. This is a named numeric vector with maximum likelihood estimates for `mu` and `sigma` and the following attributes:

`model`	The name of the model.
`density`	The density associated with the estimates.
`logLik`	The loglikelihood at the maximum.
`support`	The support of the density.
`n`	The number of observations.
`call`	The call as captured my `match.call`

References

Johnson, N. L., Kotz, S. and Balakrishnan, N. (1995) Continuous Univariate Distributions, Volume 2, Chapter 24. Wiley, New York.

See Also

Laplace for the Laplace density.

Examples

```
mllaplace(precip)
```

`mllgamma` *Log-gamma distribution maximum likelihood estimation*

Description

The maximum likelihood estimate of `shapelog` and `ratelog` are calculated by calling `mlgamma()` on the transformed data.

Usage

```
mllgamma(x, na.rm = FALSE, ...)
```

Arguments

`x`	a (non-empty) numeric vector of data values.
`na.rm`	logical. Should missing values be removed?
`...`	passed to `mlgamma`.

Details

For the density function of the log normal distribution see Loggamma.

Value

`mllgamma` returns an object of class `univariateML`. This is a named numeric vector with maximum likelihood estimates for `shapelog` and `ratelog` and the following attributes:

`model`	The name of the model.
`density`	The density associated with the estimates.
`logLik`	The loglikelihood at the maximum.
`support`	The support of the density.
`n`	The number of observations.
`call`	The call as captured my `match.call`

References

Hogg, R. V. and Klugman, S. A. (1984), Loss Distributions, Wiley.

Dutang, C., Goulet, V., & Pigeon, M. (2008). actuar: An R package for actuarial science. Journal of Statistical Software, 25(7), 1-37.

See Also

Loggamma for the log normal density.

Examples

```
mllgamma(precip)
```

`mllgser` *Logarithmic series distribution maximum likelihood estimation*

Description

For the density function of the Logarithmic series distribution see Logarithmic series. For an example data set, see corbet.

Usage

```
mllgser(x, na.rm = FALSE, ...)
```

Arguments

`x`	a (non-empty) numeric vector of data values.
`na.rm`	logical. Should missing values be removed?
`...`	Not in use.

Value

`mllgser` returns an object of class `univariateML`. This is a named numeric vector with maximum likelihood estimates for `theta`.

`model`	The name of the model.
`density`	The density associated with the estimates.
`logLik`	The loglikelihood at the maximum.
`support`	The support of the density.
`n`	The number of observations.
`call`	The call as captured my `match.call`

References

Fisher, R. A., Corbet, A. S., & Williams, C. B. (1943). The relation between the number of species and the number of individuals in a random sample of an animal population. The Journal of Animal Ecology, 12(1), 42. https://doi.org/10.2307/1411

Johnson, N. L., Kemp, A. W., & Kotz, S. (2005). Univariate Discrete Distributions (3rd ed.). Wiley-Blackwell.

See Also

Logarithmic series for the density.

Examples

```
theta_hat <- mllgser(corbet)

# The corbert data contains observations from 1 to 24.
observed <- table(corbet)

# The chi square test evaluated at the maximum likelihood is highly significant.
expected <- extraDistr::dlgser(1:24, theta_hat)
chisq.test(observed, p = expected / sum(expected))

# But chi square test evaluated at 0.997 (used in Corbet) is not.
expected <- extraDistr::dlgser(1:24, 0.997)
chisq.test(observed, p = expected / sum(expected))

# The chi square for `dzipf` is similar.
expected <- sads::dzipf(1:24, mlzipf(corbet)[1], mlzipf(corbet)[2]) * length(corbet)
chisq.test(observed, p = expected / sum(expected))
```

mlllogis *Log-logistic distribution maximum likelihood estimation*

Description

The maximum likelihood estimate of shape and rate are calculated by transforming the data back to the logistic model and applying mllogis.

Usage

```
mlllogis(x, na.rm = FALSE, ...)
```

Arguments

x	a (non-empty) numeric vector of data values.
na.rm	logical. Should missing values be removed?
...	passed to mllogis.

Details

For the density function of the log-logistic distribution see Loglogistic

Value

mllogis returns an object of class univariateML. This is a named numeric vector with maximum likelihood estimates for shape and rate and the following attributes:

model	The name of the model.
density	The density associated with the estimates.
logLik	The loglikelihood at the maximum.
support	The support of the density.
n	The number of observations.
call	The call as captured my match.call

References

Kleiber, C. and Kotz, S. (2003), Statistical Size Distributions in Economics and Actuarial Sciences, Wiley.

Klugman, S. A., Panjer, H. H. and Willmot, G. E. (2012), Loss Models, From Data to Decisions, Fourth Edition, Wiley.

Dutang, C., Goulet, V., & Pigeon, M. (2008). actuar: An R package for actuarial science. Journal of Statistical Software, 25(7), 1-37.

See Also

Loglogistic for the log-logistic density.

Examples

```
mllnorm(precip)
```

`mllnorm` *Log-normal distribution maximum likelihood estimation*

Description

The maximum likelihood estimate of `meanlog` is the empirical mean of the log-transformed data and the maximum likelihood estimate of `sdlog` is the square root of the biased sample variance based on the log-transformed data.

Usage

```
mllnorm(x, na.rm = FALSE, ...)
```

Arguments

`x`	a (non-empty) numeric vector of data values.
`na.rm`	logical. Should missing values be removed?
`...`	currently affects nothing.

Details

For the density function of the log normal distribution see Lognormal.

Value

`mllnorm` returns an object of class `univariateML`. This is a named numeric vector with maximum likelihood estimates for `meanlog` and `sdlog` and the following attributes:

`model`	The name of the model.
`density`	The density associated with the estimates.
`logLik`	The loglikelihood at the maximum.
`support`	The support of the density.
`n`	The number of observations.
`call`	The call as captured my `match.call`

References

Johnson, N. L., Kotz, S. and Balakrishnan, N. (1995) Continuous Univariate Distributions, Volume 1, Chapter 14. Wiley, New York.

See Also

Lognormal for the log normal density.

Examples

```
mllnorm(precip)
```

`mllogis` *Logistic distribution maximum likelihood estimation*

Description

Calculates the estimates using `nlm` with an exponential transform of the location parameter.

Usage

```
mllogis(x, na.rm = FALSE, ...)
```

Arguments

`x`	a (non-empty) numeric vector of data values.
`na.rm`	logical. Should missing values be removed?
`...`	currently affects nothing.

Details

For the density function of the logistic distribution see Logistic.

Value

`mllogis` returns an object of class `univariateML`. This is a named numeric vector with maximum likelihood estimates for `location` and `scale` and the following attributes:

`model`	The name of the model.
`density`	The density associated with the estimates.
`logLik`	The loglikelihood at the maximum.
`support`	The support of the density.
`n`	The number of observations.
`call`	The call as captured my `match.call`

References

Johnson, N. L., Kotz, S. and Balakrishnan, N. (1995) Continuous Univariate Distributions, Volume 2, Chapter 23. Wiley, New York.

See Also

Logistic for the Logistic density, nlm for the optimizer this function uses.

Examples

```
mllogis(precip)
```

`mllogitnorm` *Logit-Normal distribution maximum likelihood estimation*

Description

The maximum likelihood estimate of `mu` is the empirical mean of the logit transformed data and the maximum likelihood estimate of `sigma` is the square root of the logit transformed biased sample variance.

Usage

```
mllogitnorm(x, na.rm = FALSE, ...)
```

Arguments

`x`	a (non-empty) numeric vector of data values.
`na.rm`	logical. Should missing values be removed?
`...`	currently affects nothing.

Details

For the density function of the logit-normal distribution see dlogitnorm.

Value

`mllogitnorm` returns an object of class `univariateML`. This is a named numeric vector with maximum likelihood estimates for `mu` and `sigma` and the following attributes:

`model`	The name of the model.
`density`	The density associated with the estimates.
`logLik`	The loglikelihood at the maximum.
`support`	The support of the density.
`n`	The number of observations.
`call`	The call as captured my `match.call`

References

Atchison, J., & Shen, S. M. (1980). Logistic-normal distributions: Some properties and uses. Biometrika, 67(2), 261-272.

See Also

Normal for the normal density.

Examples

```
AIC(mllogitnorm(USArrests$Rape / 100))
```

mllomax *Lomax distribution maximum likelihood estimation*

Description

Uses Newton-Raphson to estimate the parameters of the Lomax distribution.

Usage

```
mllomax(x, na.rm = FALSE, ...)
```

Arguments

x	a (non-empty) numeric vector of data values.
na.rm	logical. Should missing values be removed?
...	lambda0 an optional starting value for the lambda parameter. reltol is the relative accuracy requested, defaults to .Machine$double.eps^0.25. iterlim is a positive integer specifying the maximum number of iterations to be performed before the program is terminated (defaults to 100).

Details

For the density function of the Lomax distribution see Lomax.

The likelihood estimator of the Lomax distribution is unbounded when mean(x^2) < 2*mean(x)^2. When this happens, the likelihood converges to an exponential distribution with parameter equal to the mean of the data. This is the natural limiting case for the Lomax distribution, and it is reasonable to use mlexp in this case.

Value

mllomax returns an object of class univariateML. This is a named numeric vector with maximum likelihood estimates for lambda and kappa and the following attributes:

model	The name of the model.
density	The density associated with the estimates.
logLik	The loglikelihood at the maximum.
support	The support of the density.
n	The number of observations.
call	The call as captured my match.call

References

Kleiber, Christian; Kotz, Samuel (2003), Statistical Size Distributions in Economics and Actuarial Sciences, Wiley Series in Probability and Statistics, 470, John Wiley & Sons, p. 60

See Also

Lomax for the Lomax density.

Examples

```
set.seed(3)
mllomax(extraDistr::rlomax(100, 2, 4))

# The maximum likelihood estimator may fail if the data is exponential.
## Not run:
set.seed(5)
mllomax(rexp(10))

## End(Not run)
```

`mlnaka` *Nakagami distribution maximum likelihood estimation*

Description

The maximum likelihood estimates of `shape` and `scale` are calculated by calling `mlgamma` on the transformed data.

Usage

```
mlnaka(x, na.rm = FALSE, ...)
```

Arguments

`x`	a (non-empty) numeric vector of data values.
`na.rm`	logical. Should missing values be removed?
`...`	passed to `mlgamma`.

Details

For the density function of the Nakagami distribution see Nakagami.

Value

`mlgamma` returns an object of class `univariateML`. This is a named numeric vector with maximum likelihood estimates for `shape` and `rate` and the following attributes:

`model`	The name of the model.
`density`	The density associated with the estimates.
`logLik`	The loglikelihood at the maximum.
`support`	The support of the density.
`n`	The number of observations.
`call`	The call as captured by `match.call`
`continuous`	Is the density continuous or discrete?

References

Choi, S. C, and R. Wette. "Maximum likelihood estimation of the parameters of the gamma distribution and their bias." Technometrics 11.4 (1969): 683-690.

Johnson, N. L., Kotz, S. and Balakrishnan, N. (1995) Continuous Univariate Distributions, Volume 1, Chapter 17. Wiley, New York.

See Also

Nakagami for the Nakagami distribution. GammaDist for the closely related Gamma density. See `mlgamma` for the machinery underlying this function.

Examples

```
mlgamma(precip)
```

`mlnbinom` *Negative binomial distribution maximum likelihood estimation*

Description

For the density function of the Negative binomial distribution see Negative binomial.

Usage

```
mlnbinom(x, na.rm = FALSE, ...)
```

Arguments

`x`	a (non-empty) numeric vector of data values.
`na.rm`	logical. Should missing values be removed?
`...`	The arguments `size` can be specified to only return the ml of `prob`. `reltol` is the relative accuracy requested, defaults to `.Machine$double.eps^0.25`. `iterlim` is a positive integer specifying the maximum number of iterations to be performed before the program is terminated (defaults to `100`).

Value

`mlnbinom` returns an object of class `univariateML`. This is a named numeric vector with maximum likelihood estimates for `size` and `prob` and the following attributes:

`model`	The name of the model.
`density`	The density associated with the estimates.
`logLik`	The loglikelihood at the maximum.
`support`	The support of the density.
`n`	The number of observations.
`call`	The call as captured my `match.call`

References

Johnson, N. L., Kemp, A. W., & Kotz, S. (2005). Univariate Discrete Distributions (3rd ed.). Wiley-Blackwell.

See Also

Negative binomial for the density.

Examples

```
mlnbinom(corbet)
```

`mlnorm` *Normal distribution maximum likelihood estimation*

Description

The maximum likelihood estimate of `mean` is the empirical mean and the maximum likelihood estimate of `sd` is the square root of the biased sample variance.

Usage

```
mlnorm(x, na.rm = FALSE, ...)
```

Arguments

`x`	a (non-empty) numeric vector of data values.
`na.rm`	logical. Should missing values be removed?
`...`	currently affects nothing.

Details

For the density function of the normal distribution see Normal.

Value

`mlnorm` returns an object of class `univariateML`. This is a named numeric vector with maximum likelihood estimates for `mean` and `sd` and the following attributes:

`model`	The name of the model.
`density`	The density associated with the estimates.
`logLik`	The loglikelihood at the maximum.
`support`	The support of the density.
`n`	The number of observations.
`call`	The call as captured my `match.call`

References

Johnson, N. L., Kotz, S. and Balakrishnan, N. (1995) Continuous Univariate Distributions, Volume 1, Chapter 13. Wiley, New York.

See Also

Normal for the normal density.

Examples

```
mlnorm(precip)
```

`mlparalogis` *Paralogistic distribution maximum likelihood estimation*

Description

This function estimates the only the shape parameters of the Paralogistic distribution. The rate is set to 1.

Usage

```
mlparalogis(x, na.rm = FALSE, ...)
```

Arguments

`x`	a (non-empty) numeric vector of data values.
`na.rm`	logical. Should missing values be removed?
`...`	currently affects nothing.

Details

For the density function of the Paralogistic distribution see Paralogistic.

Value

`mlparalogis` returns an object of class `univariateML`. This is a named numeric vector with maximum likelihood estimates for `shape` and the following attributes:

`model`	The name of the model.
`density`	The density associated with the estimates.
`logLik`	The loglikelihood at the maximum.
`support`	The support of the density.
`n`	The number of observations.
`call`	The call as captured my `match.call`

References

Kleiber, C. and Kotz, S. (2003), Statistical Size Distributions in Economics and Actuarial Sciences, Wiley. Klugman, S. A., Panjer, H. H. and Willmot, G. E. (2012), Loss Models, From Data to Decisions, Fourth Edition, Wiley.

See Also

Paralogistic for the paralogistic density.

Examples

```
mlparalogis(abalone$length)
```

`mlpareto` *Pareto distribution maximum likelihood estimation*

Description

The maximum likelihood estimate of `b` is the minimum of `x` and the maximum likelihood estimate of `a` is `1/(mean(log(x)) - log(b))`.

Usage

```
mlpareto(x, na.rm = FALSE, ...)
```

Arguments

`x`	a (non-empty) numeric vector of data values.
`na.rm`	logical. Should missing values be removed?
`...`	currently affects nothing.

Details

For the density function of the Pareto distribution see Pareto.

Value

`mlpareto` returns an object of class `univariateML`. This is a named numeric vector with maximum likelihood estimates for `a` and `b` and the following attributes:

`model`	The name of the model.
`density`	The density associated with the estimates.
`logLik`	The loglikelihood at the maximum.
`support`	The support of the density.
`n`	The number of observations.
`call`	The call as captured my `match.call`

References

Johnson, N. L., Kotz, S. and Balakrishnan, N. (1995) Continuous Univariate Distributions, Volume 1, Chapter 20. Wiley, New York.

See Also

Pareto for the Pareto density.

Examples

```
mlpareto(precip)
```

`mlpois` *Poisson distribution maximum likelihood estimation*

Description

The maximum likelihood estimate of `lambda` is the empirical mean.

Usage

```
mlpois(x, na.rm = FALSE, ...)
```

Arguments

`x`	a (non-empty) numeric vector of data values.
`na.rm`	logical. Should missing values be removed?
`...`	currently affects nothing.

Details

For the density function of the Poisson distribution see Poisson.

Value

`mlpois` returns an object of class `univariateML`. This is a named numeric vector with maximum likelihood estimates for `lambda` and the following attributes:

`model`	The name of the model.
`density`	The density associated with the estimates.
`logLik`	The loglikelihood at the maximum.
`support`	The support of the density.
`n`	The number of observations.
`call`	The call as captured my `match.call`

References

Johnson, N. L., Kemp, A. W., & Kotz, S. (2005). Univariate Discrete Distributions (3rd ed.). Wiley-Blackwell.

See Also

Poisson for the Poisson density.

Examples

```
mlpois(ChickWeight$weight)
```

`mlpower` *Power distribution maximum likelihood estimation*

Description

The maximum likelihood estimate of `alpha` is the maximum of `x + epsilon` (see the details) and the maximum likelihood estimate of `beta` is `1/(log(alpha)-mean(log(x)))`.

Usage

```
mlpower(x, na.rm = FALSE, ...)
```

Arguments

`x`	a (non-empty) numeric vector of data values.
`na.rm`	logical. Should missing values be removed?
`...`	`epsilon` is a positive number added to `max(x)` as an to the maximum likelihood. Defaults to `.Machine$double.eps^0.5`.

Details

For the density function of the power distribution see PowerDist. The maximum likelihood estimator of `alpha` does not exist, strictly speaking. This is because `x` is supported `c(0, alpha)` with an open endpoint on alpha in the `extraDistr` implementation of `dpower`. If the endpoint was closed, `max(x)` would have been the maximum likelihood estimator. To overcome this problem, we add a possibly user specified `epsilon` to `max(x)`.

Value

`mlpower` returns an object of class `univariateML`. This is a named numeric vector with maximum likelihood estimates for `alpha` and `beta` and the following attributes:

`model`	The name of the model.
`density`	The density associated with the estimates.
`logLik`	The loglikelihood at the maximum.

support	The support of the density.
n	The number of observations.
call	The call as captured my match.call

References

Arslan, G. "A new characterization of the power distribution." Journal of Computational and Applied Mathematics 260 (2014): 99-102.

See Also

PowerDist for the power density. extraDistr::Pareto for the closely related Pareto distribution.

Examples

```
mlpower(precip)
```

---

mlrayleigh *Rayleigh distribution maximum likelihood estimation*

Description

Calculates the sigma parameter as the square root of half the empirical second moment.

Usage

```
mlrayleigh(x, na.rm = FALSE, ...)
```

Arguments

x	a (non-empty) numeric vector of data values.
na.rm	logical. Should missing values be removed?
...	currently affects nothing.

Details

For the density function of the Rayleigh distribution see Rayleigh.

Value

mlrayleigh returns an object of class univariateML. This is a named numeric vector with maximum likelihood estimates for sigma and the following attributes:

model	The name of the model.
density	The density associated with the estimates.
logLik	The loglikelihood at the maximum.
support	The support of the density.
n	The number of observations.
call	The call as captured my match.call

References

Johnson, N. L., Kotz, S. and Balakrishnan, N. (1995) Continuous Univariate Distributions, Volume 1, Chapter 18. Wiley, New York.

See Also

Rayleigh for the Rayleigh density.

Examples

```
mlrayleigh(precip)
```

`mlsged` *Skew Generalized Error distribution maximum likelihood estimation*

Description

Joint maximum likelihood estimation as implemented by fGarch::sgedFit.

Usage

```
mlsged(x, na.rm = FALSE, ...)
```

Arguments

`x`	a (non-empty) numeric vector of data values.
`na.rm`	logical. Should missing values be removed?
`...`	currently affects nothing.

Details

For the density function of the Student t-distribution see sged.

Value

`mlsged` returns an object of class `univariateML`. This is a named numeric vector with maximum likelihood estimates for the parameters `mean`, `sd`, `nu`, `xi`, and the following attributes:

`model`	The name of the model.
`density`	The density associated with the estimates.
`logLik`	The loglikelihood at the maximum.
`support`	The support of the density.
`n`	The number of observations.
`call`	The call as captured my `match.call`

References

Nelson D.B. (1991); Conditional Heteroscedasticity in Asset Returns: A New Approach, Econometrica, 59, 347<U+2013>370.

Fernandez C., Steel M.F.J. (2000); On Bayesian Modelling of Fat Tails and Skewness, Preprint.

See Also

sged for the Student t-density.

Examples

```
mlsged(precip)
```

`mlsnorm` *Skew Normal distribution maximum likelihood estimation*

Description

Joint maximum likelihood estimation as implemented by fGarch::snormFit.

Usage

```
mlsnorm(x, na.rm = FALSE, ...)
```

Arguments

`x`	a (non-empty) numeric vector of data values.
`na.rm`	logical. Should missing values be removed?
`...`	currently affects nothing.

Details

For the density function of the Student t distribution see dsnorm.

Value

`mlsnorm` returns an object of class `univariateML`. This is a named numeric vector with maximum likelihood estimates for the parameters `mean`, `sd`, `xi` and the following attributes:

`model`	The name of the model.
`density`	The density associated with the estimates.
`logLik`	The loglikelihood at the maximum.
`support`	The support of the density.
`n`	The number of observations.
`call`	The call as captured my `match.call`

References

Fernandez C., Steel M.F.J. (2000); On Bayesian Modelling of Fat Tails and Skewness, Preprint.

See Also

dsnorm for the Student-t density.

Examples

```
mlsnorm(precip)
```

`mlsstd` *Skew Student t-distribution maximum likelihood estimation*

Description

Joint maximum likelihood estimation as implemented by fGarch::sstdFit.

Usage

```
mlsstd(x, na.rm = FALSE, ...)
```

Arguments

`x`	a (non-empty) numeric vector of data values.
`na.rm`	logical. Should missing values be removed?
`...`	currently affects nothing.

Details

For the density function of the skew Student t-distribution see sstd.

Value

`mlsstd` returns an object of class `univariateML`. This is a named numeric vector with maximum likelihood estimates for the parameters `mean`, `sd`, `nu`, `xi` and the following attributes:

`model`	The name of the model.
`density`	The density associated with the estimates.
`logLik`	The loglikelihood at the maximum.
`support`	The support of the density.
`n`	The number of observations.
`call`	The call as captured my `match.call`

References

Fernandez C., Steel M.F.J. (2000); On Bayesian Modelling of Fat Tails and Skewness, Preprint.

See Also

sstd for the Skew Student t-density.

Examples

```
mlsstd(precip)
```

`mlstd` *Student-t distribution maximum likelihood estimation*

Description

Joint maximum likelihood estimation as implemented by fGarch::stdFit.

Usage

```
mlstd(x, na.rm = FALSE, ...)
```

Arguments

`x`	a (non-empty) numeric vector of data values.
`na.rm`	logical. Should missing values be removed?
`...`	currently affects nothing.

Details

For the density function of the Student t-distribution see std.

Value

`mlstd` returns an object of class `univariateML`. This is a named numeric vector with maximum likelihood estimates for the parameters `mean`, `sd`, `nu` and the following attributes:

`model`	The name of the model.
`density`	The density associated with the estimates.
`logLik`	The loglikelihood at the maximum.
`support`	The support of the density.
`n`	The number of observations.
`call`	The call as captured my `match.call`

References

Johnson, N. L., Kotz, S. and Balakrishnan, N. (1995) Continuous Univariate Distributions, Volume 1, Chapter 13. Wiley, New York.

See Also

std for the Student-t density.

Examples

```
mlstd(precip)
```

`mlunif` *Uniform distribution maximum likelihood estimation*

Description

The estimates are `min(x)` and `max(x)`.

Usage

```
mlunif(x, na.rm = FALSE, ...)
```

Arguments

`x`	a (non-empty) numeric vector of data values.
`na.rm`	logical. Should missing values be removed?
`...`	currently affects nothing.

Details

For the density function of the logistic distribution see Uniform.

Value

`mlunif` returns an object of class `univariateML`. This is a named numeric vector with maximum likelihood estimates for `min` and `max` and the following attributes:

`model`	The name of the model.
`density`	The density associated with the estimates.
`logLik`	The loglikelihood at the maximum.
`support`	The support of the density.
`n`	The number of observations.
`call`	The call as captured my `match.call`

References

Johnson, N. L., Kotz, S. and Balakrishnan, N. (1995) Continuous Univariate Distributions, Volume 2, Chapter 26. Wiley, New York.

See Also

Uniform for the uniform density.

Examples

```
mlunif(precip)
```

`mlweibull` *Weibull distribution maximum likelihood estimation*

Description

For the density function of the Weibull distribution see Weibull.

Usage

```
mlweibull(x, na.rm = FALSE, ...)
```

Arguments

`x`	a (non-empty) numeric vector of data values.
`na.rm`	logical. Should missing values be removed?
`...`	`shape0` is an optional starting value for the `shape` parameter. `reltol` is the relative accuracy requested, defaults to `.Machine$double.eps^0.25`. `iterlim` is a positive integer specifying the maximum number of iterations to be performed before the program is terminated (defaults to `100`).

Details

Uses mlgumbel to estimate the parameters of the Weibull distribution.

Value

`mlweibull` returns an object of class `univariateML`. This is a named numeric vector with maximum likelihood estimates for `shape` and `scale` and the following attributes:

`model`	The name of the model.
`density`	The density associated with the estimates.
`logLik`	The loglikelihood at the maximum.
`support`	The support of the density.
`n`	The number of observations.
`call`	The call as captured my `match.call`

References

Johnson, N. L., Kotz, S. and Balakrishnan, N. (1995) Continuous Univariate Distributions, Volume 1, Chapter 21. Wiley, New York.

See Also

Weibull for the Weibull density.

Examples

```
BIC(mlweibull(precip))
```

`mlzip` *Zero-inflated Poisson distribution maximum likelihood estimation*

Description

For the density function of the zeero-inflated Poisson distribution see Zero-inflated Poisson distribution.

Usage

```
mlzip(x, na.rm = FALSE, ...)
```

Arguments

`x`	a (non-empty) numeric vector of data values.
`na.rm`	logical. Should missing values be removed?
`...`	Not currently in use.

Value

`mlzip` returns an object of class `univariateML`. This is a named numeric vector with maximum likelihood estimates for `lambda` and `pi` and the following attributes:

`model`	The name of the model.
`density`	The density associated with the estimates.
`logLik`	The loglikelihood at the maximum.
`support`	The support of the density.
`n`	The number of observations.
`call`	The call as captured my `match.call`

References

Johnson, N. L., Kemp, A. W., & Kotz, S. (2005). Univariate Discrete Distributions (3rd ed.). Wiley-Blackwell.

See Also

Zero-inflated Poisson distribution for the density.

Examples

```
mlzip(corbet)
```

`mlzipf` *Zipf distribution maximum likelihood estimation*

Description

For the density function of the Zipf distribution see Zipf.

Usage

```
mlzipf(x, na.rm = FALSE, ...)
```

Arguments

`x`	a (non-empty) numeric vector of data values.
`na.rm`	logical. Should missing values be removed?
`...`	Not currently in use.

Details

This function follows the same format as every other function in the package, but most applications of Zipf's law use rank-abundance data. See, e.g., sads::fitzipf for estimation of this sort of data.

Value

`mlzipf` returns an object of class `univariateML`. This is a named numeric vector with maximum likelihood estimates for `N` and `s` and the following attributes:

`model`	The name of the model.
`density`	The density associated with the estimates.
`logLik`	The loglikelihood at the maximum.
`support`	The support of the density.
`n`	The number of observations.
`call`	The call as captured my `match.call`

References

Johnson, N. L., Kemp, A. W., & Kotz, S. (2005). Univariate Discrete Distributions (3rd ed.). Wiley-Blackwell.

See Also

Zipf for the density.

Examples

```
AIC(mlzipf(corbet)) # 2729.536
AIC(mllgser(corbet)) # 2835.883
```

`model_select` *Fit multiple models and select the best fit*

Description

Selects the best model by log-likelihood, aic, or bic.

Usage

```
model_select(
  x,
  models = univariateML_models,
  criterion = c("AIC", "BIC", "logLik"),
  na.rm = FALSE,
  type = c("both", "discrete", "continuous"),
  return = c("best", "all"),
  ...
)
```

Arguments

`x`	a (non-empty) numeric vector of data values.
`models`	a character vector containing the distribution models to select from; see `print(univariateML_models)`. Defaults to all implemented models.
`criterion`	the model selection criterion. Must be one of `"AIC"`, `"BIC"`, and `"logLik"`, ignoring case. Defaults to `"AIC"`.
`na.rm`	logical. Should missing values be removed?
`type`	Either `"both"`, `"discrete"`, or `"continuous"`. The supplied `models` vector is restricted to the desired class.
`return`	character length 1. "univariateML" (default) if the function should return the single best model; "all" if a tibble data frame of all results should be returned, sorted by decreasing model performance.
`...`	unused.

Value

The return value depends on the `return` argument. For `return = "best"` (default), `model_select` returns an object of class `univariateML`

For `return = "all"`, `model_select` returns a tibble data frame with the following columns:

`model`	The name of the model.

\verb{d_loglik, d_aic, d_bic}
: See `loglik, aic, bic`.

`p`
: Number of parameters fitted.

\verb{loglik, aic, bic}
: The negative log-likelihood at the maximum, the aic, and the bic, respectively. The minimum of each of these is noted and then subtracted from each value to give their delta versions `d_loglik, d_aic, d_bic`

. So, the model with the lowest aic will have `d_aic` of 0; the `d_aic` of all the other models shows how much higher their aics are from the minimum. The same goes with `d_loglik` and `d_bic`.

`ml`
: The internal code name for the model.

`univariateML`
: The `univariateML` object for the model. This is `return = "all"`, this object is returned for all tested models.

See Also

Johnson, N. L., Kotz, S. and Balakrishnan, N. (1995) Continuous Univariate Distributions, Volume 1, Chapter 17. Wiley, New York.

Examples

```
# Select among all possible continuous models.
model_select(precip, type = "continuous")

# View possible models to fit.
print(univariateML_models)

# Try out only gamma, Weibull, and exponential.
model_select(precip, c("gamma", "weibull", "exp"))

# Fit the discrete `corbet` data to all available discrete models
model_select(corbet, type = "discrete", return = "all")
```

---

plot.univariateML *Plot, Lines and Points Methods for Maximum Likelihood Estimates*

Description

The `plot`, `lines`, and `points` methods for `univariateML` objects.

Usage

```
## S3 method for class 'univariateML'
plot(x, range = NULL, kind = c("d", "p", "q"), ...)

## S3 method for class 'univariateML'
```

```
lines(x, range = NULL, kind = c("d", "p", "q"), ...)

## S3 method for class 'univariateML'
points(x, range = NULL, kind = c("d", "p", "q"), ...)
```

Arguments

x	a univariateML object.
range	range of x values to plot, i.e. c(lower, upper).
kind	can be density, probability, or quantile.
...	parameters passed to plot, lines, or points.

Value

An invisible copy of x.

Examples

```
plot(mlweibull(datasets::precip), main = "Annual Precipitation in US Cities")
lines(mlgamma(datasets::precip), lty = 2)
rug(datasets::precip)
```

ProbabilityPlots *Probability Plots Using Maximum Likelihood Estimates*

Description

Make quantile-quantile plots and probability-probability plots using maximum likelihood estimation.

Usage

```
ppmlplot(y, obj, plot.it = TRUE, datax = FALSE, ...)

ppmlline(...)

ppmlpoints(y, obj, plot.it = TRUE, datax = TRUE, ...)

qqmlplot(y, obj, plot.it = TRUE, datax = FALSE, ...)

qqmlline(y, obj, datax = FALSE, probs = c(0.25, 0.75), qtype = 7, ...)

qqmlpoints(y, obj, plot.it = TRUE, datax = TRUE, ...)
```

Arguments

y	Numeric vector; The data to plot on the y axis when datax is FALSE.
obj	Either an univariateML object or a function that returns a univariateML object when called with y as its only argument.
plot.it	Logical; should the result be plotted?
datax	Logical; should y be plotted on the x-axis? Defaults to FALSE in qqmlplot and ppmlplot but TRUE in qqmlpoints and ppmlpoints.
...	Graphical parameters.
probs	Numeric vector of length two, representing probabilities. Corresponding quantile pairs define the line drawn.
qtype	The type of quantile computation used in quantile.

Details

qqmlplot produces a quantile-quantile plot (Q-Q plot) of the values in y with respect to the distribution defined by obj, which is either a univariateML object or a function returning a univariateML object when called with y. qqmlline adds a line to a <U+201C>theoretical<U+201D>, quantile-quantile plot which passes through the probs quantiles, by default the first and third quartiles. qqmlpointsbehaves like stats::points and adds a Q-Q plot to an existing plot.

ppmlplot, ppmlline, and ppmlpoints produce probability-probability plots (or P-P plots). They behave similarly to the quantile-quantile plot functions.

This function is modeled after qqnorm.

Quantile-quantile plots and probability-probability plots are only supported for continuous distributions.

Graphical parameters may be given as arguments to all the functions below.

Value

For qqmlplot, qqmlpoints, ppmlplot, and ppmlpoints, a list with components x (plotted on the x axis) and y (plotted on the y axis). qqmlline and ppmlline returns nothing.

References

M. B. Wilk, R. Gnadadesikan, Probability plotting methods for the analysis for the analysis of data, Biometrika, Volume 55, Issue 1, March 1968, Pages 1<U+2013>17, https://doi.org/10.1093/biomet/55.1.1

Examples

```
## Make a single probability plot with a line.

obj <- mlgamma(Nile)
qqmlplot(Nile, obj)
qqmlline(Nile, obj)

## Make multiple probability plots. datax = TRUE must be used to make this
## look good.
```

```
ppmlplot(airquality$Wind, mlgamma, main = "Many P-P plots")
ppmlpoints(airquality$Wind, mlexp, col = "red")
ppmlpoints(airquality$Wind, mlweibull, col = "purple")
ppmlpoints(airquality$Wind, mllnorm, col = "blue")
```

`univariateML_construct` *Construct* `univariateML` *object.*

Description

Construct `univariateML` object.

Usage

```
univariateML_construct(estimates, name, params)
```

Arguments

`estimates`	The estimated parameters
`name`	Name of the `ml***` function.
`params`	List of `loglik`, `call`, and `n`.

Value

Object of class `univariateML`

`univariateML_metadata` *Metadata for* `univariateML` *models.*

Description

Metadata for `univariateML` models.

Usage

```
univariateML_metadata
```

Format

An object of class `list` of length 42.

`univariateML_models` *Implemented models*

Description

Vector of all supported models in `univariateML`.

Usage

```
univariateML_models
```

Format

An object of class `character` of length 42.

Details

The currently supported models are mlbeta, mlbetapr, mlbinom, mlburr, mlcauchy, mldunif, mlexp, mlfatigue, mlgamma, mlged, mlgeom, mlgompertz, mlgumbel, mlinvburr, mlinvgamma, mlinvgauss, mlinvweibull, mlkumar, mllaplace, mllgamma, mllgser, mlllogis, mllnorm, mllogis, mllogitnorm, mllomax, mlnaka, mlnbinom, mlnorm, mlparalogis, mlpareto, mlpois, mlpower, mlrayleigh, mlsged, mlsnorm, mlsstd, mlstd, mlunif, mlweibull, mlzip, mlzipf

Index